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EFFECT OF APPROACH-FLOW VELOCITY AND TEMPERATURE NONUNIFORMITIES ON BOUNDARY-LAYER FLOW AND HEAT TRANSFER

E. M. SPARROW, R. M. ABDEL-WAHED and S. V. PATANKAR
Department of Mechanical Engineering, University of Minnesota, Minneapolis, Minnesota, U.S.A.

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Abstract—The laminar flow and heat transfer on a flat plate subjected to nonuniform velocity and temperature profiles in the approaching free stream have been analyzed. The plate is situated in the laminar wake of an upstream plate. The extent of the approach-flow nonuniformities depends on the streamwise length of the wake relative to the length of the upstream plate. It was found that the effect of the nonuniformities is to reduce the wall shear and heat transfer on the downstream plate relative to their values for a uniform approach flow. Reductions of up to fifty percent were encountered. The extent of the reductions diminishes with increasing downstream distance, but non-negligible effects persist to a considerable length along the plate.

NOMENCLATURE

D, drag force;
L, length of upstream plate, Fig. 1;
Pr, Prandtl number;
Q, overall heat-transfer rate;
q, local heat flux;
S, spacing between plates, Fig. 1;
T, temperature;
T_w, wall temperature;
T_∞, temperature of stream approaching upstream plate;
U_∞, velocity of stream approaching upstream plate;
u, v, velocity components;
x, y, coordinates.

Greek symbols

ν, kinematic viscosity;
ρ, density;
τ, wall shear stress.

Subscript

0, approach flow with uniform velocity and temperature.

INTRODUCTION AND BACKGROUND

IN THE analysis of flat plate boundary layers, it is usual to assume that the velocity and temperature of the upstream approach flow are uniform. This model is appropriate to situations where there are no upstream surfaces (e.g. plates, cylinders) to distort the otherwise uniform flow and temperature distributions. On the other hand, when such upstream surfaces are present, there may be substantial nonuniformities of velocity and temperature in the flow approaching the leading edge of the flat plate. Such nonuniformities will affect the velocity and thermal boundary-layer development on the plate, with corresponding effects on the wall shear and heat transfer.

In this paper, we are concerned with laminar flow and heat transfer on a flat plate in a hydrodynamically and thermally nonuniform approach flow. Specific consideration is given to a plate situated in the laminar wake of an upstream plate, as pictured in the lower portion of Fig. 1. As shown there, a fluid with uniform approach velocity U_∞ and temperature T_∞ flows past an upstream plate of length L. Downstream of this plate, there is a gap followed by a second plate of indefinite length. The spacing between the two plates is S.

The temperature T_w of the first plate differs from T_∞, so that the presence of the plate induces a thermal boundary layer as well as a velocity boundary layer. The velocity and temperature profiles associated with these boundary layers are modified by convection and diffusion in the wake. Consequently, the extent of the profile nonuniformity in the flow arriving at the second plate depends on the streamwise length of the wake region. In general, the longer the wake region, the more uniform are the profiles in the arriving flow.

The main concern of this study is to determine the hydrodynamic and thermal response of the second plate (whose temperature is also T_w) to the aforementioned profile nonuniformities. To facilitate the identification of the effect of the nonuniformities, the heat-transfer and skin friction results are plotted as ratios with their counterparts for a flow having uniform approach-flow profiles (i.e. Blasius flow). These ratios are independent of the Reynolds number but are dependent on the relative spacing S/L between the plates. By examining the departures of these ratios from unity, the region of the plate that is affected by the approach-flow profile nonuniformities can be determined.

The complexity of the problem precludes similarity-type solutions. A finite-difference method was employed to obtain the solutions and results, which depend on the geometric parameter S/L and on the

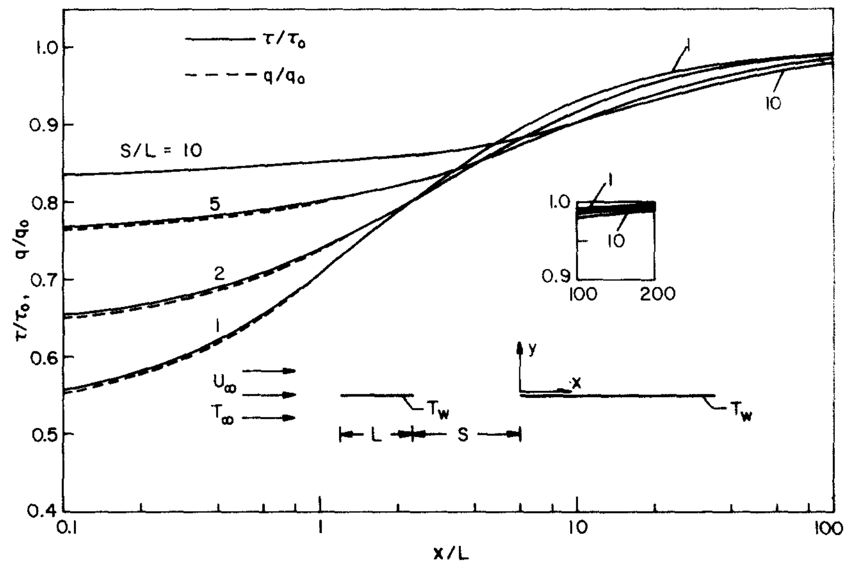

FIG. 1. Lower portion: schematic diagram of the flow configuration. Graph: response of the local wall shear and local heat flux to nonuniformities in the approach flow.

Prandtl number (which was assigned a value of 0.7). The finite difference computations extended over the first plate and the wake, as well as the second plate.

It does not appear that boundary-layer problems of the type considered here have been previously treated in the literature. It is, however, relevant to take note of previous studies of the velocity field in the laminar wake downstream of a flat plate. In the early analyses (summarized in [1] and [2]), the wake flow was assumed to obey the conventional boundary layer equations (i.e. Goldstein model). More recently, it has been demonstrated that this simple boundary layer model is not able to provide an accurate description of the flow in the immediate neighborhood of the trailing edge of the plate (references [3] and [4] are representative of an extensive literature). On the other hand, with increasing distance downstream from the trailing edge, the results from the simple model become increasingly more accurate.

By employing the results of [4] the accuracy of the wake velocities from the simple boundary-layer model can be estimated. The downstream distance from the trailing edge required to attain 2% accuracy in the velocity at the wake centerline ($y = 0$) can be calculated from

$$\frac{S}{L} \sim \frac{47}{(U_\infty L/\nu)^{3/8}} \tag{1}$$

where $U_\infty L/\nu$ is the plate Reynolds number. For Reynolds numbers of 10^4 and 10^5, the respective S/L values are about 1.5 and 0.6. At larger S/L, progressively higher accuracies are attained. Furthermore, the accuracies at off-centerline locations are higher than that at the centerline.

Since the wake velocity field is actually peripheral to the main focus of the present research, it is reasonable to employ as simple a model for the wake as is consistent with acceptable accuracy for the approach flow at the second plate. In view of the foregoing discussion, it appears that the conventional boundary-layer equations are entirely satisfactory provided that consideration is given to spacings $S/L \geqq 1$. This approach has been adopted here.

ANALYSIS AND SOLUTIONS

The analysis is based on the incompressible boundary-layer equations for x-momentum (without pressure gradient) and energy, plus the continuity equation. When these equations are recast in suitable dimensionless form, two parameters are encountered, namely, the relative separation distance S/L between the plates and the Prandtl number Pr. The usual no-slip and impermeability conditions, $u = v = 0$, are imposed at the surfaces $y = 0$ of both the upstream and downstream plates, whereas $\partial u/\partial y = v = 0$ along the wake centerline $y = 0$ due to symmetry. The surface temperature of both plates is T_w, and along the wake centerline $\partial T/\partial y = 0$. At the edge of the boundary layer, $u \to U_\infty$ and $T \to T_\infty$.

The solution of the problem was carried out numerically by the use of Patankar–Spalding method [5]. This solution procedure involves step-by-step marching in the streamwise direction. In the present problem, the marching was initiated at the leading edge of the upstream plate and was continued along the length of that plate, through the wake, and along the length of the downstream plate.

Special provisions had to be made to accommodate the processes which occur near the leading edge of the downstream plate. In that region, a new boundary layer develops within the already existing, relatively thick boundary layer spawned by the upstream plate and the wake. The need for special treatment of this situation arises because the deployment of the grid points across the boundary layer is automatically tailored by the solution method to accommodate the thick boundary layer in the flow approaching the downstream plate. As a consequence, there would be too few grid points near the wall to accurately describe the development of a new, relatively thin boundary layer. Additional grid points were, therefore, implanted in the wall region of the downstream plate, and computational experiments were performed to ensure that the results were insensitive to the number of such points and their deployment.

Velocity and temperature solutions were carried out for relative plate spacings S/L of 1, 2, 5 and 10, with the Prandtl number being fixed at 0.7. This variation of spacing gives rise to significant changes in the velocity and temperature profiles in the flow approaching the downstream plate. The velocity profiles in the approach flow (i.e. at $x = 0$) are illustrated in the upper left panel of Fig. 2 for the extremes of $S/L = 1$ and 10. As is shown there, the effect of elongating the wake region is not only to flatten the velocity profile but also to thicken the boundary layer. Thus, whereas the velocity at $y = 0$ for the $S/L = 10$ case is substantially larger than that for the $S/L = 1$ case (the respective u/U_∞ values are 0.877 and 0.644), the situation is reversed at larger y. The temperature profiles at $x = 0$, when represented in terms of $(T-T_w)/(T_\infty-T_w)$, are very similar to the velocity profiles at $x = 0$.

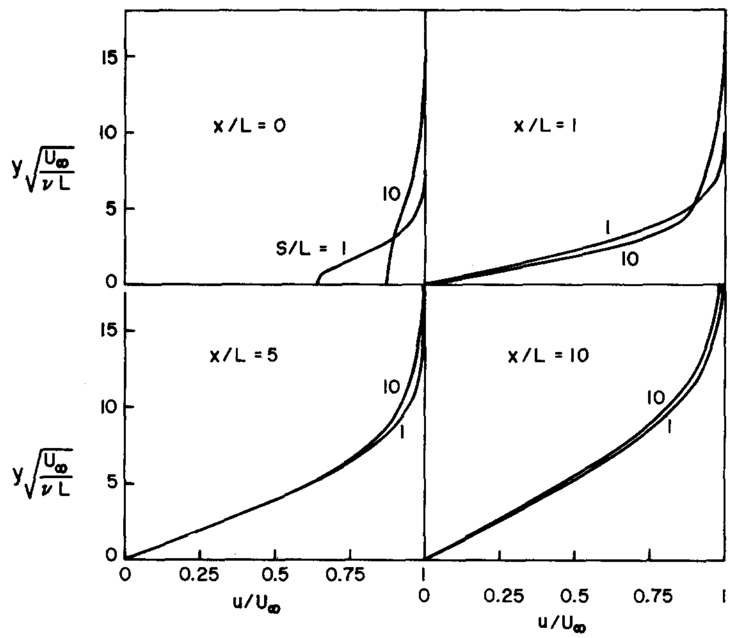

FIG. 2. Evolution of the velocity profiles along the downstream plate.

RESULTS AND DISCUSSION

The response of the skin friction and heat transfer on the downstream plate to the just-discussed approach-flow nonuniformities will now be examined. The numerical solutions provide values of the local wall shear stress τ and local heat flux q as a function of position along the plate. These τ and q values will be compared to those for a conventional flat plate boundary layer where the velocity and temperature in the approach flow is uniform. The wall shear and heat flux corresponding to the uniform approach flow may be denoted by τ_0 and q_0, where

$$\tau_0/\rho U_\infty^2 = 0.332/(U_\infty x/\nu)^{1/2},$$
$$q_0 x/k(T_w - T_\infty) = 0.293(U_\infty x/\nu)^{1/2}. \quad (2)$$

With these, the ratios τ/τ_0 and q/q_0 may be evaluated, with both numerator and denominator corresponding to the same Reynolds number and the same streamwise location x.

The τ/τ_0 and q/q_0 ratios are plotted in Fig. 1 as a function of the dimensionless streamwise coordinate x/L, with S/L as curve parameter. The wall shear and heat flux results are respectively represented by solid and dashed lines. It is seen from the figure that the effect of the approach-flow nonuniformities is to reduce the wall shear and heat flux relative to their values for a uniform approach flow. The greatest reductions occur just downstream of the leading edge (i.e. at small x), where τ/τ_0 and q/q_0 values as low as about 0.5 are in evidence. With increasing downstream distance, τ/τ_0 and q/q_0 increase toward unity, as the memory of the approach-flow nonuniformities wanes. The close proximity of the solid and dashed lines indicates that the wall shear and heat flux ratios are affected to about the same extent, which is in keeping with the fact that the temperature solutions correspond to a Prandtl number near unity.

Figure 1 contains certain noteworthy findings. One of these is the streamwise extent of the plate that is influenced by the approach-flow nonuniformities. The region where τ and q are affected by more than 5% extends from $x/L = 0$ to 15–35 (depending on S/L). Effects that exceed 2% are encountered between $x/L = 0$ to 40–100. Therefore, it appears that the memory of the initial nonuniformities lingers for a considerable distance along the plate.

The behavior of the results for the various spacing ratios is also worthy of note. At small values of x/L, the τ/τ_0 and q/q_0 curves are arranged in monotonic order with S/L, proceeding from lower to upper as S/L increases. With increasing downstream distance, the curves cross, so that for $x/L > \sim 10$, a new monotonic ordering is established which is reversed from that which initially prevailed.

In appraising this behavior, it is relevant to examine the evolution of the velocity and temperature profiles along the plate. Figure 2 shows the velocity profiles at $x/L = 0$ (approach flow), 1, 5 and 10 for $S/L = 1$ and 10 (the temperature profiles exhibit a similar trend and are not shown). Just upstream of the leading edge, $x = 0$ and small y, it is seen that smaller velocities are associated with lower values of S/L. It is, therefore, reasonable that smaller values of τ/τ_0 and q/q_0 should correspond to smaller values of S/L, at least near the leading edge.

With increasing downstream distance, the initial relationships between the velocity (and temperature) profiles for the various S/L are markedly modified by the growing boundary layer, as can be seen in Fig. 2. In particular, in the near-wall region, larger velocities are ultimately associated with lower values of S/L. It is the rearrangement of the near-wall velocities which brings about the change in the ordering of the curves of Fig. 1 as a function of S/L.

Results have also been obtained for the drag D and overall heat-transfer rate Q defined as

$$D = \int_0^x \tau \, dx, \quad Q = \int_0^x q \, dx \tag{3}$$

per unit span. In order to avoid possible inaccuracies associated with the integration of τ and q at small x values, D and Q were respectively evaluated by differencing the momentum and energy fluxes at $x = x$ with those at $x = 0$. The quantities D_0 and Q_0 were obtained by direct integration of equation (2). The results are presented in terms of the ratios D/D_0 and Q/Q_0, where D_0 and Q_0 pertain to a uniform approach flow. The D/D_0 and Q/Q_0 ratios are plotted in Fig. 3 as a function of x/L, and the curves are parameterized by S/L. As expected, the most significant effects of the approach-flow nonuniformity are in evidence at small and moderate x/L. As x/L increases, the curves tend to increase toward $D/D_0 = Q/Q_0 = 1$, but the rate of approach is quite slow. Thus, even at $x/L = 200$, D/D_0 and Q/Q_0 are less than 0.95 for all the spacings S/L that were investigated. The influence of the initial nonuniformities on the drag and overall heat-transfer rate is, therefore, rather far reaching.

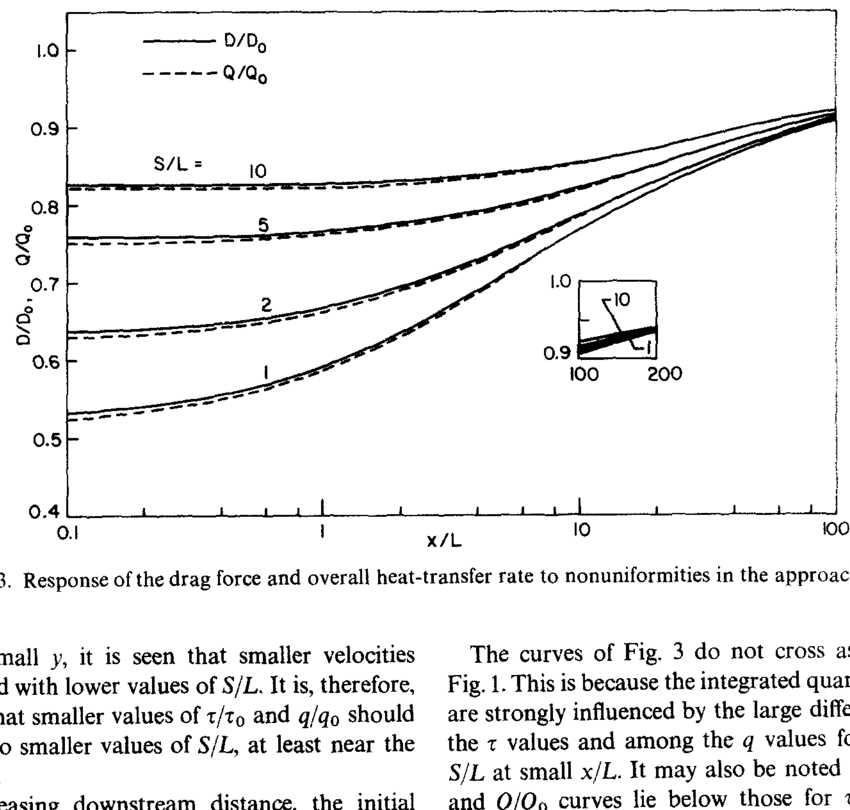

FIG. 3. Response of the drag force and overall heat-transfer rate to nonuniformities in the approach flow.

The curves of Fig. 3 do not cross as do those of Fig. 1. This is because the integrated quantities of Fig. 3 are strongly influenced by the large differences among the τ values and among the q values for the various S/L at small x/L. It may also be noted that the D/D_0 and Q/Q_0 curves lie below those for τ/τ_0 and q/q_0, which is consistent with the fact that the latter take on their smallest values at small x/L and increase with increasing downstream distance.

CONCLUDING REMARKS

The results of the present analysis have demonstrated that the presence of velocity and temperature nonuniformities in the approach flow can have a significant effect on the wall shear and heat transfer in a flat plate boundary-layer flow. Even for moderate nonuniformities, reductions in shear and heat flux of up to 50% relative to those for a uniform approach flow were encountered. The extent of the reductions diminishes with increasing downstream distance, but non-negligible effects persist to a considerable length along the plate.

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EFFETS DE LA NON-UNIFORMITE DE VITESSE ET DE TEMPERATURE DANS L'ECOULEMENT D'APPROCHE SUR L'ECOULEMENT DE COUCHE LIMITE ET SUR LE TRANSFERT THERMIQUE

Résumé—On étudie l'écoulement laminaire et le transfert thermique sur une plaque plane soumise à des profils non uniformes de vitesse et de température dans l'écoulement libre en amont. La plaque est située dans le sillage laminaire d'une autre plaque. L'étendue des non-uniformités de l'écoulement d'approche dépend de la longueur du sillage relativement à la longueur de la plaque d'amont. Il est constaté que l'effet des non-uniformités est de réduire le frottement pariétal et le transfert de chaleur sur la plaque d'aval, par rapport au cas d'un écoulement d'attaque uniforme. On a observé des réductions jusqu'à cinquante pour cent. L'importance des réductions diminue quand la distance en aval augmente, mais des effets non négligeables persistent pour une longueur considérable le long de la plaque.

DER EINFLUSS VON NICHTGLEICHFÖRMIGKEITEN DER GESCHWINDIGKEIT UND DER TEMPERATUR IN DER ANSTRÖMUNG AUF DIE GRENZSCHICHTSTRÖMUNG UND DEN WÄRMEÜBERGANG

Zusammenfassung—Für den Fall ungleichförmiger Geschwindigkeits- und Temperaturprofile in der Anströmung wird die laminare Strömung und der Wärmeübergang an einer ebenen Platte untersucht. Die Platte befindet sich im laminaren Abströmbereich einer anderen stromaufwärts gelegenen Platte. Der Ungleichförmigkeitsgrad der Anströmung hängt vom Verhältnis der Länge des Totwassergebietes zur Länge der stromaufwärts gelegenen Platte ab. Infolge der Ungleichförmigkeit werden sowohl die Wandschubspannung wie der Wärmeübergang vermindert, wobei Verringerungen bis zu 50% auftreten. In Strömungsrichtung nehmen diese Verminderungseffekte ab, sie sind jedoch über eine beträchtliche Plattenlänge als nicht vernachlässigbar anzusehen.

ВЛИЯНИЕ СКОРОСТИ НАБЕГАЮЩЕГО ПОТОКА И НЕОДНОРОДНОСТИ ТЕМПЕРАТУРЫ НА ТЕЧЕНИЕ В ПОГРАНИЧНОМ СЛОЕ И ТЕПЛООБМЕН

Аннотация — В статье рассматривается ламинарное течение и теплообмен на плоской пластине при неоднородных профилях скорости и температуры в набегающем потоке. Пластина помещена в ламинарный след другой пластины, расположенной выше по течению. Степень неоднородности набегающего потока зависит от отношения длины следа к длине пластины, расположенной вверх по течению. Найдено, что неоднородность снижает касательное напряжение и коэффициент теплообмена пластины, расположенной в следе, по сравнению со значениями для однородного набегающего потока. Иногда это снижение достигает 50%. Степень этого влияния уменьшается с увеличением расстояния вниз по потоку, однако продолжает сказываться на значительном расстоянии вдоль пластины.